Computational Modelling of Concrete at Early Ages using DIANA

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1 INTRODUCTION

At the TU München the research on concrete structures that are sensible to cracking at early ages has a long tradition. Intensive work has been done on concrete technology to minimise the cracking risk due to the loss of the heat of hydration and this research still goes on at the Chair for Building Materials. At the Chair for Analysis of Structures we try to predict the crack width in the cases where cracking is inevitable. DIANA provides a wide set of features that are necessary for this kind of calculation and offers the possibility to add even more through user-supplied-subroutines. This article describes some basic problems and our solutions, that may be useful for others analysing structures under early temperature load.

2 OVERVIEW

For those not familiar with the problem of early age cracking, here a (very) short overview: The hydration process does mainly two things to concrete – heat production and hardening. Unfortunately this combination leads to tension, if the structure can't follow the temperature movements freely during the cooling phase. When the tensile stresses exceed the tensile strength, cracking occurs. This is followed by the loss of water tightness, if the crack width gets to big. Besides trying to avoid cracking through optimisation of the concrete properties (what sometimes is simply not possible), another solution is to limit the crack width by reinforcement. Because the design according to most codes is a general approach, the amount of reinforcement can be reduced significantly in some cases by examining the hydration and cracking process with a transient Finite-Element analysis for a given structure more closely.

Typical examples of structures, that are sensitive to early age cracking are inner linings of tunnels, mass concrete structures, structures with parts of different ages and members, that are restrained by friction. We investigated tunnel linings some time ago and achieved results that lead to a new reduced requirement for the minimum reinforcement for Munich subway tunnels [10]. This article will concentrate on base slabs of water-tight concrete constructions, that are often built in Munich due to a high ground water level.

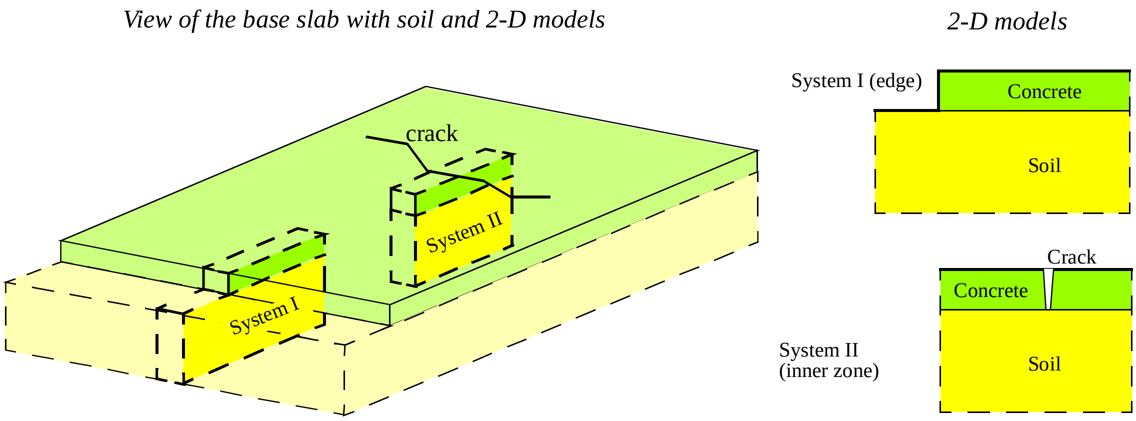


Fig 1: Analysis of base slabs with 2-D models (plain strain).

3 COUPLED ANALYSIS

The starting point for a calculation is to decide, what kind of coupling between temperature and stress has to be taken into account. It is widely accepted that it is accurate to calculate the fields of temperature and hydration in a first step, and then apply these data on a transient stress analysis. There are two ways in DIANA to achieve this kind of coupling: Either through the built in *coupled analysis*, or by creating tables of temperature and degree of reaction after the heat transfer calculation and then read these tables for a separate static analysis. The last way is favourable in combination with a phased analysis. While the temperatures are treated as loads in the static analysis, the degree of reaction is the main parameter for the development of the concrete properties. The degree of reaction *r* is introduced as the ratio of the amount of heat at time t to the total amount of hydration heat:

$$r(t) = Q(t)/Q_\infty \qquad (1)$$

The heat production q(r) is the time-derivative of Q(t) and has to be specified as a polynomial function of r for input in DIANA:

$$q(t) = \frac{dQ}{dt} \text{ and } q(r) = \dot{r} \cdot Q_\infty \qquad (2)$$

It should be mentioned here, that there is a supplemental application called HYMOSTRUC [13] that can be used together with DIANA to estimate the heat production of a concrete from its ingredients. In this article it will be assumed that either experimental data or an analytic formula (e.g. according to [12]) are used to get the function of heat production q(r). The phenomenon of faster reactions at higher temperature is modelled by an Arrhenius function internally. Because DIANA uses a concept according to Reinhardt et al. [9], it actually starts its calculation at time 0 with r = 0.1. This means, that already 10% of the heat of hydration are gone. To calculate the right temperatures, the state at time 0 has to be the temperature distribution after 10% of hydration, what isn't correct either because the degree of hydration isn't distributed uniformly over the structure. We avoided this problem by manipulating our input so that r = 0.1 is the starting point of the hydration process (cf. fig. 2). The ”real” degree of hydration is then called α.

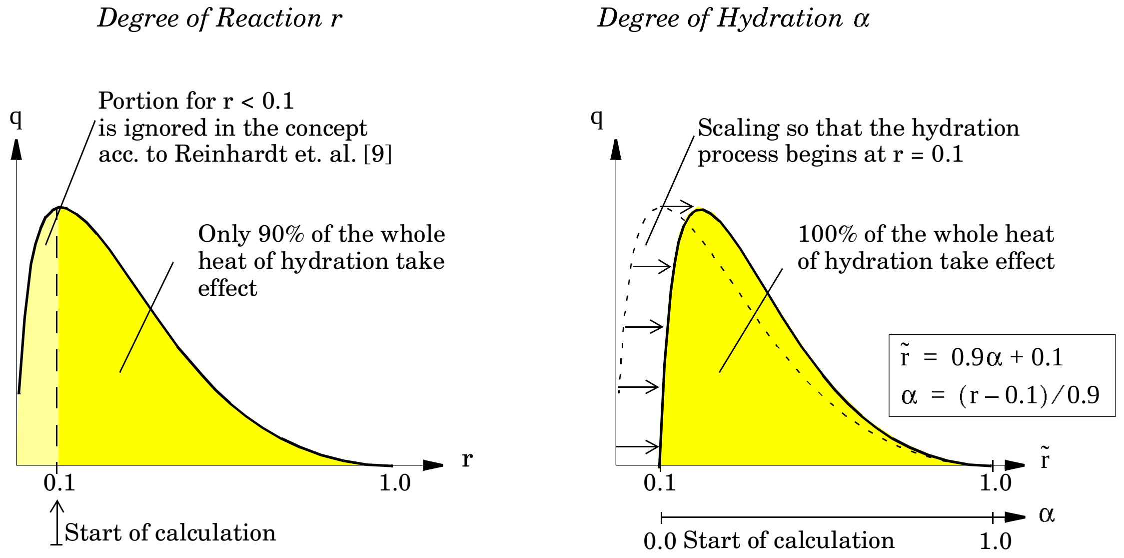


Fig. 2: Manipulation of the degree of reaction.

The time steps have to be small enough during the “hot phase” of the heat production, in order to model the high gradients in heat production during the first 48 hours. Depending on the concrete characteristics, we used time steps of 1 or 2 hours. Afterwards longer time steps are adequate. Fig. 3 shows the temperature development over time in different layers of a slab. The soil has to be introduced in the heat transfer calculation because it acts as a heat storage medium. With boundary conditions at the lower side of the slab, this effect would be neglected.

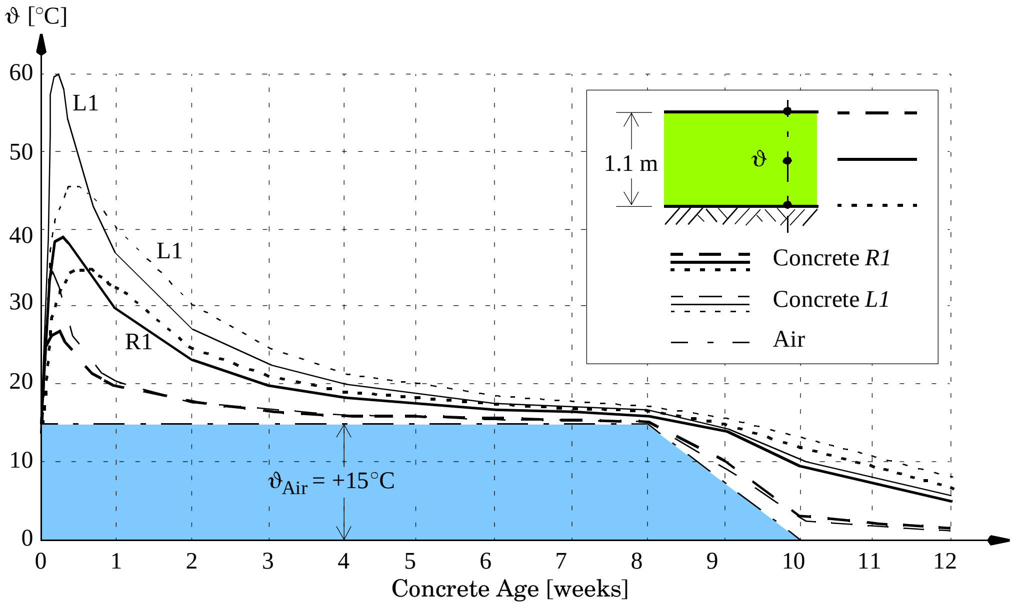


Fig. 3: Temperature development for two different concrete compositions in characteristic layers of the slab

4 STRUCTURAL ANALYSIS

The calculated temperatures and degrees of reaction for each time step are transferred as input to the structural analysis, where the calculation starts again at the beginning. The same discretisation in space is used for the heat analysis and the structural analysis. The mesh is generated for the 8-nodes plane-strain elements used in the structural analysis (element type CQ16E). Nevertheless DIANA uses 4-nodes elements internally for the heat analysis to ensure compatibility of the temperature strains with the interpolation functions used in the structural analysis.

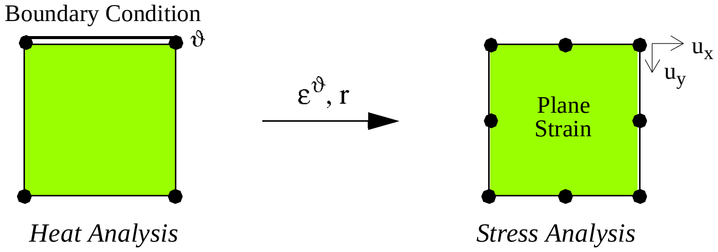


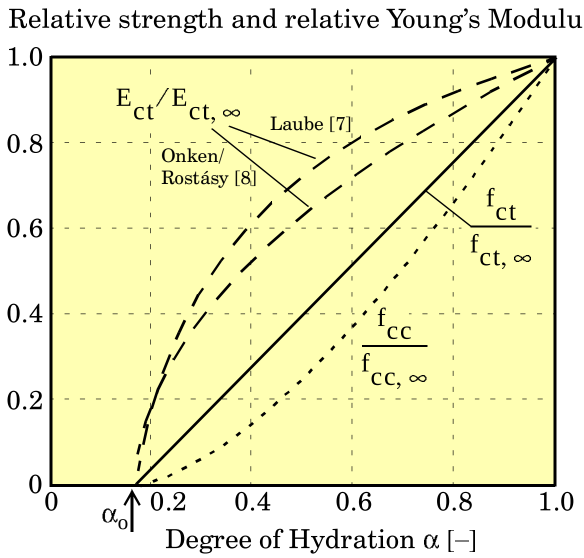
Fig. 4: Elements for coupled analysis

The discretisation in time may be different in the structural analysis. As the development of the mechanical properties begins at a degree of hydration between 0.1 and 0.35, there usually can be one big step in time for the first 12 h at the beginning. During the rapid evolution of the mechanical properties, the time steps have to be relatively small (2 h) and may be elongated after 2-3 days. It is reasonable to choose the time steps according to the heat analysis until the first crack occurs. To achieve convergence the time steps must be adjusted carefully during cracking. To get accurate results for the crack width under temperature restraint it is absolutely important to ensure, that only one discrete crack opens within one time step. If multiple cracks open simultaneously the crack spacing and in consequence the crack width would be underestimated.

The tension softening reduces the stresses due to temperature restraint and new cracks are caused by stresses in the reinforcement, that depend on the crack width. Therefore it is necessary to control the opening of the first crack in a homogenous stressed structure through a slightly lower value for the tensile strength. Further cracks open successively due to the ongoing increase of the temperature load and the development of the stresses around the first crack. This leads to a clear sequence of crack openings during the calculation, where usually new cracks occur in the middle between two older cracks.

5 MATERIAL MODELLS

There exists a complete and well certified set of concrete properties formulated in dependence of the degree of hydration [8, 12]. We used these definitions instead of the built in functions through user-supplied-subroutines (fig 5).



$$\frac{f_{cc}(\alpha)}{f_{cc,\,\alpha\,=\,1}} = \left(\frac{\alpha-\alpha_o}{1-\alpha_o}\right)^{3/2} \geq 0 \qquad [8]$$

$$\frac{f_{ct}(\alpha)}{f_{ct,\,\alpha\,=\,1}} = \frac{\alpha-\alpha_o}{1-\alpha_o} \geq 0 \qquad [8]$$

$$\frac{E(\alpha)}{E_{\alpha\,=\,1}} = \left(\frac{\alpha-\alpha_o}{1-\alpha_o}\right)^{2/3} \geq 0 \qquad [8]$$

$$G_f(\alpha) = G_{f,\,\infty}\frac{E(\alpha)}{E_{\alpha\,=\,1}} \qquad [11]$$

Fig. 5: Development of the properties of early age concrete

The discrete reinforcement was modelled with truss elements. To obtain a crack width in accordance with the design code, we developed an element for the intersection of the reinforcement with the discrete crack. This element is derived from the formulas for the crack width in the design codes and thus guarantees conformity with the code [4, 10]. It also models the whole bond-slip behaviour of the reinforcement, so that all other elements were connected without bond-slip. The element was realised by an user-supplied subroutine USRMAT.

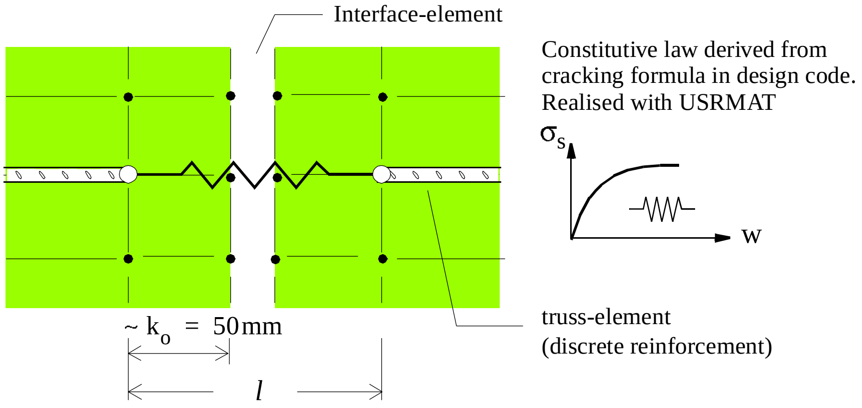


Fig. 6: User-defined element for the crossing of the reinforcement with the discrete crack.

We also made use of interface elements to model the frictional interaction between base concrete and soil. We started with a high elastic stiffness for the interface-elements. This lead to convergence problems until we reduced the stiffness to a lower value, that was nevertheless stiff enough compared to the soil itself.

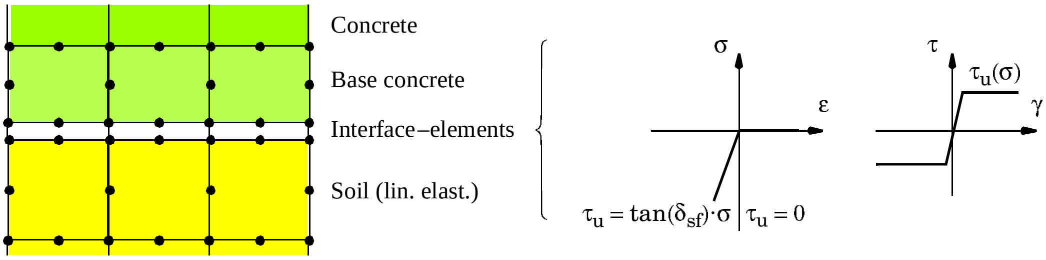


Fig. 7: Modelling of the joint soil-concrete

6 RESULTS

The results of our calculations are shown by some examples to demonstrate the possibilities of the described analysis procedures. The distribution of the temperature and of the degree of reaction at the edge of the slab after 2 days of hydration are shown in Fig. 8. The difference in hydration speed between the surface and the interior of the slab is caused by different temperature histories and therefore different effective concrete ages at the same physical age.

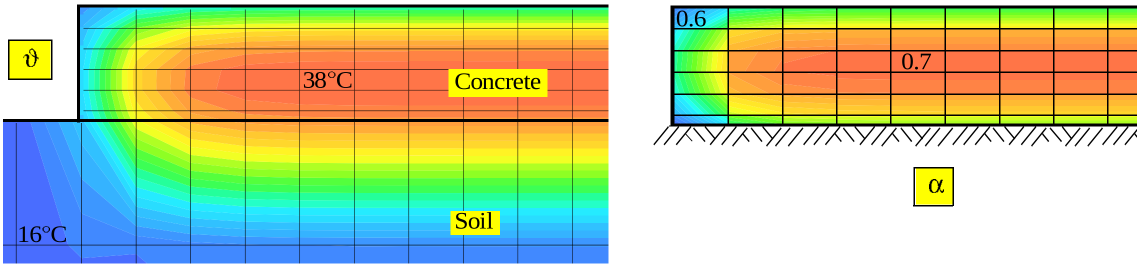


Fig. 8: Distribution of the temperature and the degree of hydration after 2 days of hydration

Fig. 9 shows a typical example for the evolution of the restraint stresses. The tensile strength is reached in the centre of the slab after 10 days in this example (a value that depends strongly on the boundary conditions). The crack opens like a zipper over the whole cross-section. The development of the crack-width is shown in Fig. 10 together with a displacement plot after 28 days.

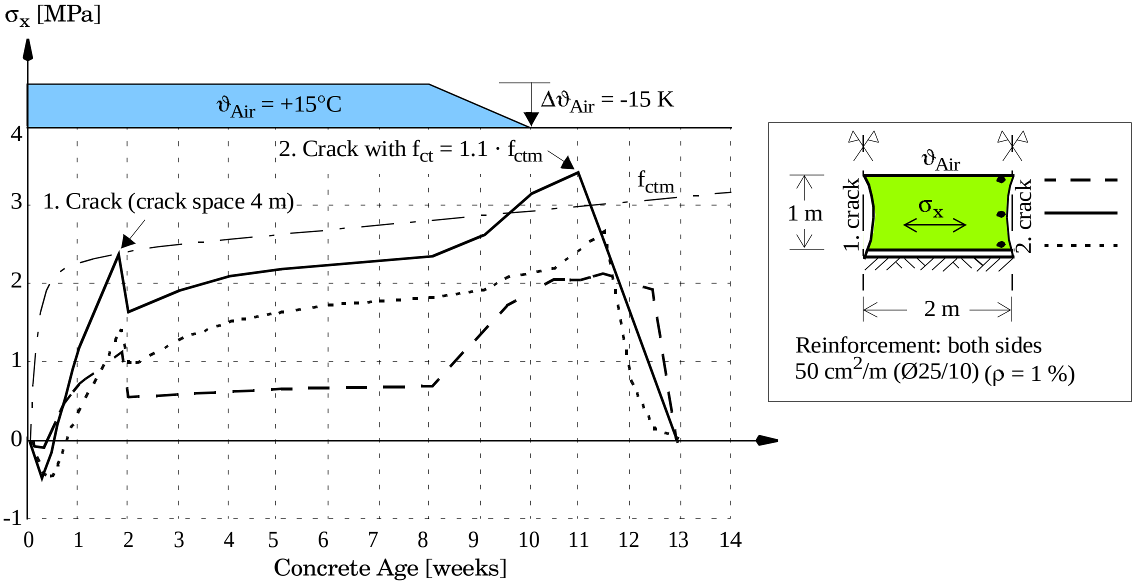


Fig. 9: Development of the restraint stresses and the tensile strength.

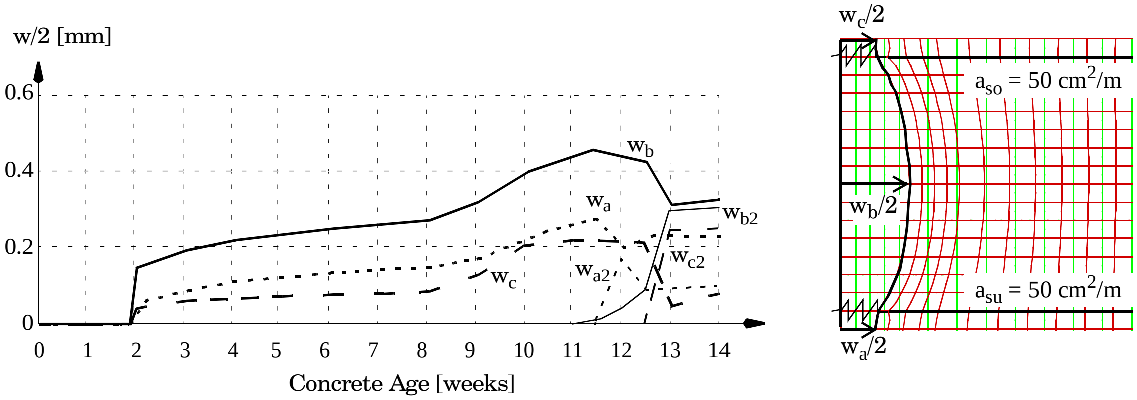


Fig. 10: Development of the crack width and displacement plot after 28 days.

7 CONCLUSIONS

DIANA offers most features that are necessary to calculate temperature and stress fields in hydrating concrete. Some built-in functions, like the maturity influence on the Young’s modulus or the degree of reaction concept must be used with care. For those properties the extensibility through user-supplied subroutines is one of the strongest features of DIANA.

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